ALGEBRA FINAL EXAMINATION - BACKPAPER

This exam is of **50 marks** and is **3 hours long**. Please **read all the questions carefully**. Please feel free to use whatever theorems you have learned in class after stating them clearly. Please sign the following statement:

I have not used any unfair or illegal means to answer any of the questions in this exam.

Name: **Signature:**

1. Let R be a ring with 1. Show that a unitary R-module I is injective if and only if for every left ideal L of R, any R-module homomorphism $\phi : L \longrightarrow I$ can be extended to an R-module homomorphism $\tilde{\phi} : L \longrightarrow I$. (10)

2. Show that a commutative ring with identity R is local $\Leftrightarrow$ for all $r, s \in R$ such that $r + s = 1$ implies r or s is a unit. (10)

3. Let I be a non-zero ideal in a PID R. Show that R/I is both Artinian and Noetherian. (10)

4. State whether the following are **True** or **False**. If **True**, prove it. If **False**, give a counter-example. In all questions R is a commutative ring with 1.

- Let A and B be R modules. Then $A \otimes_R B = A \otimes_{\mathbb{Z}} B$. (5)
- $a, b \in R$. $a \otimes b = a \otimes c \Rightarrow b = c$. (5)
- If P is an injective module over R then it is free. (5)
- Every exact sequence of $\mathbb{Z}$-modules is split. (5)